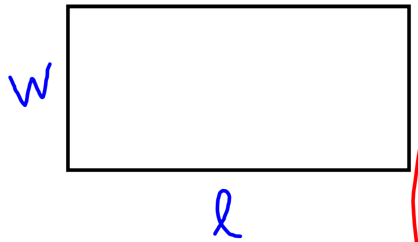
A rectangle has a perimeter of 100 ft. What length and width should it have so the area is a max?



der. = 0

$$2l + 2w = 100$$

$$2w = 100 - 2l$$

$$w = 50 - l$$

$$A = lw$$

Equation or Formula

$$A = l(50 - l)$$

$$A = 50l - l^2$$

derivative $\longrightarrow$ $A' = 50 - 2l$

max $A' = 0$

$$50 - 2l = 0$$

$$l = 25\text{ ft}$$

$$w = 25\text{ ft}$$

Sometimes Max. Area $= (25\text{ ft})^2 = 625\text{ sq. ft.}$

Find two positive numbers whose product is 192 and whose sum is a minimum.

$xy = 192$

$x = \frac{192}{y}$

$y + \frac{192}{y} = S$

$S = y + 192y^{-1}$

$S' = 1 + -192y^{-2}$

$S' = 1 + \frac{-192}{y^2}$

$\boxed{1 + \frac{-192}{y^2}} = 0$

$\frac{y^2 - 192}{y^2} = 0$

$y^2 - 192 = 0$

$y^2 = 192$

$y = \sqrt{192}$

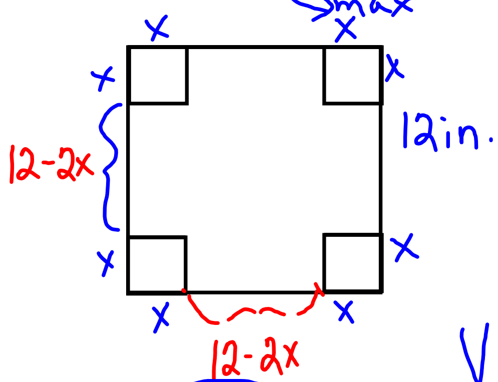
$x = \frac{192}{y}$

$x = \frac{192}{\sqrt{192}} \cdot \frac{\sqrt{192}}{\sqrt{192}} = \frac{\cancel{192}\sqrt{192}}{\cancel{192}}$

$x = \sqrt{192}$

An open box is to be made from a square piece of material, 12in on each side, by cutting equal squares from each corner and turning up the sides. Find the volume of the largest box that can be made.

max



$V = lwh$

$V = (12-2x)(12-2x)x$

$V = (144 - 48x + 4x^2)x$

$V = 144x - 48x^2 + 4x^3$

$V' = 144 - 96x + 12x^2$

$144 - 96x + 12x^2 = 0$

$12 - 8x + x^2 = 0$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$\cancel{x = 6}$ or 2

extraneous

$l = 12-2x = 8\text{in.}$

$w = 12-2x = 8\text{in}$

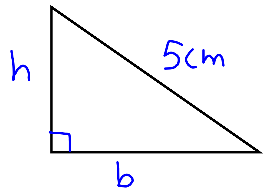
$h = x = 2\text{in.}$

Max. Volume

$V = (8\text{in})(8\text{in})(2\text{in})$

$V = 128\text{in}^3$

What is the largest possible area for a right triangle whose hypotenuse is 5 cm long. And what are its dimension



$$h^2 + b^2 = 25$$

$$b^2 = 25 - h^2$$

$$b = \sqrt{25-h^2}$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}\left(\sqrt{25-h^2}\right)\cdot h$$

$$A = \frac{h}{2}\left(25-h^2\right)^{\frac{1}{2}}$$

$$A = \frac{h}{2}\sqrt{25-h^2}$$

$$A' = \frac{h}{2}\cdot\frac{1}{\cancel{2}}\left(25-h^2\right)^{-\frac{1}{2}}\cdot -\cancel{2}h +$$

$$A' = \boxed{\frac{-h^2}{2\sqrt{25-h^2}} + \frac{\sqrt{25-h^2}\cdot\sqrt{25-h^2}}{2\sqrt{25-h^2}}}\sqrt{25-h^2}\cdot\frac{1}{2}$$

$$\frac{-h^2 + 25 - h^2}{2\sqrt{25-h^2}} = 0$$

$$-2h^2 + 25 = 0$$

$$-2h^2 = -25$$

$$h^2 = \frac{25}{2}$$

$$h = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}}$$

$$b = \frac{5}{\sqrt{2}}$$

$$b = \sqrt{25-h^2}$$

$$b = \sqrt{25-\frac{25}{2}}$$

$$b = \sqrt{\frac{50-25}{2}} = \sqrt{\frac{25}{2}}$$

$$b = \frac{5}{\sqrt{2}}$$

$$A = \frac{1}{2}\cdot\frac{5}{\sqrt{2}}\cdot\frac{5}{\sqrt{2}}$$

$$A = \frac{25}{4}\text{ in}^2$$

$$\sqrt{25-b^2}$$

$$\sqrt{9-4} = \sqrt{5}$$

$$\neq \sqrt{9} - \sqrt{4}$$

$$3-2=1$$

$y = 2x \qquad y' = 2$

$y = \frac{1}{3}x \qquad y' = \frac{1}{3}$

$y = \frac{x}{3} \qquad y' = \frac{1}{3}$

$y = \frac{h}{2} \qquad y' = \frac{1}{2}$

HW p. 214

1, 3, 4, 6, 7, 9